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Numerical study of plasma–wall transition in an oblique magnetic field

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Abstract

The interaction of a plasma with a fixed wall is investigated numerically. The ions are described by a kinetic model, while the electrons are assumed to be at thermal equilibrium. Finite Debye length effects are taken into account. An Eulerian code is used for the ion dynamics, which enables us to obtain a fine resolution of both position and velocity space. First, we analyse the effect of ionization and collisions, which bring the ion flow to supersonic velocity at the entrance of the Debye sheath (Bohm's criterion). Second, we consider a collisionless sheath with an oblique magnetic field. A magnetic presheath, which has a width of several ion gyroradii, is located between the Debye sheath and the bulk plasma. We perform a systematic numerical study of these sheaths for different incidences of the magnetic field.

Keywords: Magnetic sheath; Vlasov equation; Simulation

1. Introduction and model equations

A detailed understanding of the dynamics of magnetized plasmas in contact with a wall is of great importance in fusion physics, particularly for the design of divertor plates and for the correct description of probes in tokamaks. Pioneering results relative to plasma–wall interactions are due to Chodura [1], who has performed numerical studies using a particle-in-cell code (PIC). The present work describes numerical results obtained with an Eulerian code [2], which displays a lower level of noise, even in regions of low plasma density, such as the Debye sheath in the immediate vicinity of the wall. We also extend Chodura's results, which were obtained in a collisionless regime, to include the effect of collisions and ionization.

We study the interaction of a plasma with a perfectly absorbing wall, in the presence of an oblique magnetic field. The magnetic field, which is spatially uniform and constant in time, lies in the (x, y) plane and makes an angle α with the y axis (Fig. 1). We use a one-dimensional model in space – only the variations along the x coordinate, normal to the wall, are taken into account – but the three components of the velocity are included. The phase space is, therefore, four-dimensional (x, v_x, v_y, v_z).

The ions are described by the Vlasov equation, with a Bhatnagar–Gross–Krook (BGK) term

$$\frac{\partial f_i}{\partial t} + v_x \frac{\partial f_i}{\partial x} + \frac{e}{m_i}(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \frac{\partial f_i}{\partial \mathbf{v}} = v(f_M - f_i), \qquad (1)$$

where $f_i(x, \mathbf{v}, t)$ is the ion distribution function in the sheath and $f_M(\mathbf{v})$ is the equilibrium ion distribution in the bulk plasma. The latter is assumed to be Maxwellian with temperature T_{io}. v is the collision and ionization rate. The ion density is $n_i(x,t) = \int\int\int f_i(x, \mathbf{v}, t) \, d^3v$.

Assuming Boltzmann distributed electrons, their density is given by

$$n_e(x,t) = n_0 \exp(e\phi(x,t)/k_B T_e), \qquad (2)$$

where k_B is the Boltzmann's constant, ϕ the electrostatic potential and n_0 is the uniform equilibrium density in the bulk plasma. Finally, the system is closed by Poisson's equation

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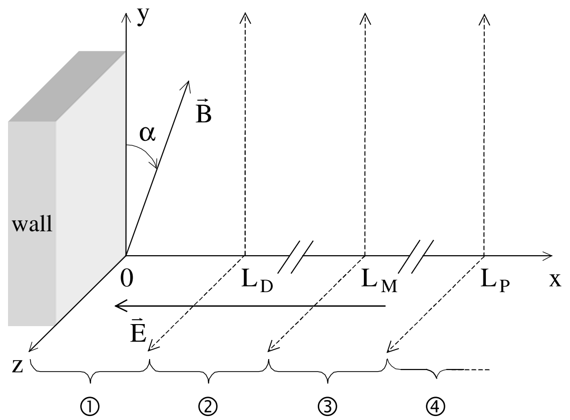

Fig. 1. Geometry of the sheath. Debye (electrostatic) sheath (1), magnetic presheath (2), collisional and ionizing presheath (3) and the bulk plasma (4).

$$\frac{\partial^2 \phi(x,t)}{\partial x^2} = -\frac{e}{\varepsilon_0}(n_i(x,t) - n_e(x,t)). \quad (3)$$

The Vlasov equation is solved numerically with an Eulerian code [3,4], which is based on a regular mesh covering the entire four-dimensional phase space. The ion distribution function is defined as a smooth function of the phase space variables. The time integration relies on a splitting scheme, which treats each phase space direction separately. Poisson's equation is solved with a finite-difference scheme. Due to the non-linearity introduced by the Boltzmann relation (2), an iterative procedure is used.

2. Numerical results

2.1. Collisional and ionizing sheath without magnetic field

The dynamics of the physical system described in the previous section can be characterized by a relatively small number of dimensionless parameters. Here, we have the ion to electron mass ratio ($m_i/m_e = 3672$ for a deuterium plasma), the ion to electron temperature ratio at the entrance of the sheath ($T_{io}/T_e = 5$), the ratio of the ion cyclotron frequency to the ion plasma frequency (ω_{ci}/ω_{pi}, equal to zero in this section) which measures the strength of the magnetic field, and finally the ratio of the collision–ionization rate to the ion plasma frequency ($\nu/\omega_{pi} = 0.1$). With the previous values of the dimensionless parameters, the ion mean free path is $\lambda \approx 22\lambda_{De}$, where λ_{De} is the Debye length computed at the electron temperature.

Eqs. (1)–(3) give a complete, self-consistent description of the sheath. Although we are primarily interested in stationary solutions of this system, in practice we solve the time-dependent problem numerically, and wait until a steady state has emerged. Therefore, one needs to specify both initial and boundary conditions. Concerning the ion distribution function, we assume a perfectly absorbing wall, with zero inward flux at $x = 0$. At the boundary with the bulk plasma ($x = L_p$), we take a Maxwellian boundary distribution ($f_b = f_M$). The actual position of this boundary is somewhat arbitrary, and is chosen so that the spatial variations become negligible. The initial condition is spatially uniform and equal to f_b. We also need boundary conditions on the electrostatic potential, and fix $e\phi/k_B T_e = 1.82$ on the left (wall) and $e\phi/k_B T_e = 0$ on the right (plasma). This choice means that we are polarizing the wall with a negative potential with respect to the bulk plasma. This value of the potential at the wall is chosen arbitrarily, but is consistent with the floating potential observed in experiments (of the order of a few $k_B T_e/e$).

Figs. 2–5 show the results of the computer simulations with $\mathbf{B} = 0$. Near the wall, we observe a space charge in a layer of 10 λ_{De} (Fig. 2), which defines the Debye sheath. The other region (between the Debye sheath and the bulk plasma) is the so-called presheath (7–8 mean-free-paths thick), where the plasma is quasi-neutral and collisions and ionization dominate. Without a magnetic field, the phase space is two-dimensional (x, v_x) and is presented in Fig. 3. The ion distribution is mainly modified for positive velocities. Cross-cuts of f_i at different positions (Fig. 4) show that the velocity distribution is narrower near the wall, which means that the ion temperature is not constant. Besides, the shape is no longer Maxwellian.

Fig. 5 presents the ion current and mean ion velocity perpendicular to the wall. The latter is increased from zero in the bulk plasma, up to the sound speed $c_s = ((T_e + T_{io})/m_i)^{1/2}$ at the wall. The standard (fluid)

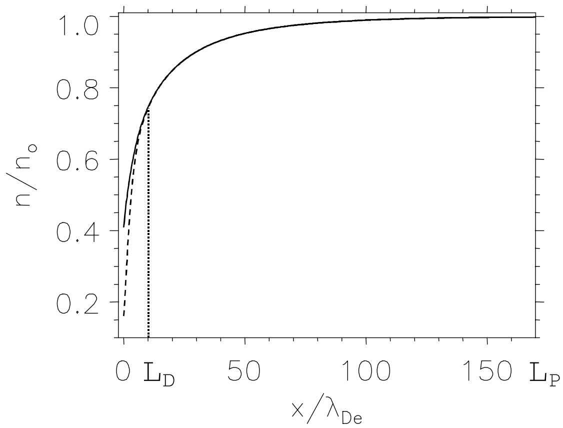

Fig. 2. Ion (solid line) and electron (dashed line) densities normalized to the bulk plasma density n_0 (no magnetic field). L_D: entrance of Debye sheath; L_P: bulk plasma.

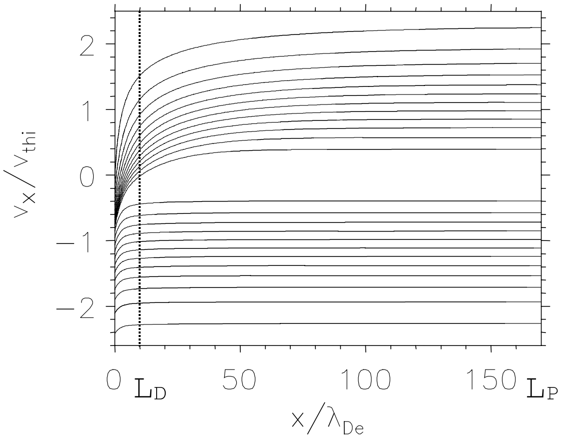

Fig. 3. Phase space: contours of constant level of the ion distribution function in the (x, v_x) plane (no magnetic field). L_D: entrance of Debye sheath; L_P: bulk plasma.

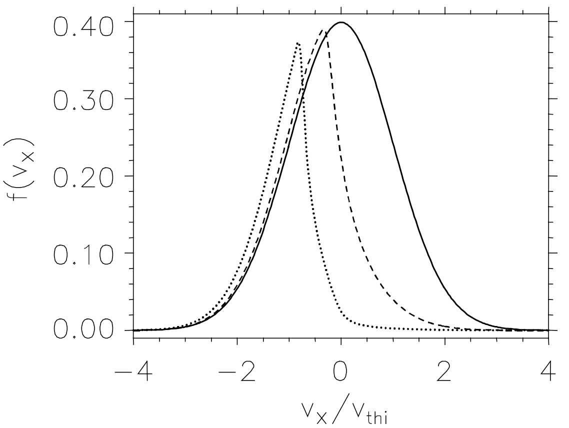

Fig. 4. Ion velocity distribution $f(v_x)$ at $x = 0$ (wall, dotted line), $x = 20\ \lambda_{De}$ (intermediate position, dashed line) and $x = 170\ \lambda_{De}$ (Maxwellian bulk plasma, solid line). (No magnetic field).

Bohm's criterion would require that $v_x = c_s$ at the entrance of the electrostatic Debye sheath, where quasineutrality breaks down. However, in our case, a kinetic version of Bohm's criterion [5,6] should be used, which requires that $m_i \langle v_x^{-2} \rangle^{-1} \geqslant k_B T_e$ (angular brackets denote averaging with the ion distribution f_i). Although the term $\langle v_x^{-2} \rangle$ is not easy to compute numerically, since it can give rise to a divergence, we have estimated that the kinetic Bohm's criterion is roughly satisfied in the vicinity of the wall.

2.2. Collisionless sheath with an oblique magnetic field

In the absence of collisions and ionization, one has $\nu/\omega_{pi} = 0$. We take $\omega_{ci}/\omega_{pi} = 0.1$ (roughly consistent

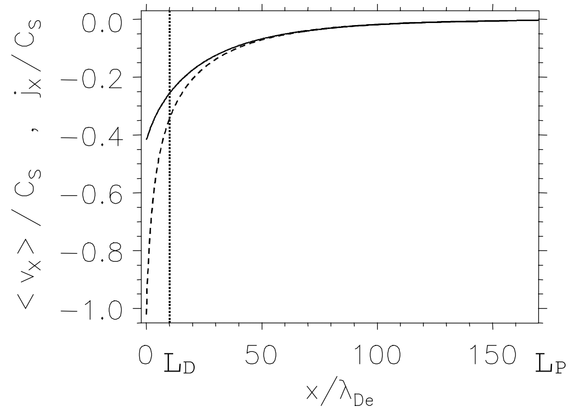

Fig. 5. Mean ion velocity v_x (dashed line) and current j_x (solid line) normalized to the sound speed c_s (no magnetic field). L_D: entrance of Debye sheath; L_P: bulk plasma.

with the values observed in a tokamak edge plasma), and keep the same values for the remaining dimensionless parameters. Another interesting parameter, which can be expressed in terms of those previously defined, is the ratio of the (ion and electron) gyro-radii to the electron Debye length. In the present case, we have $\rho_i/\lambda_{De} = 22.4$ and $\rho_e/\lambda_{De} = 0.16$. This latter value shows that the electrons are strongly magnetized; we further assume that they are thermalized along the magnetic field lines, so that their density is still given by the Boltzmann relation (2). On the other hand, the ions are weakly magnetized, and a full kinetic treatment is necessary.

Since, in the absence of collisions and ionization, the ions cannot be accelerated by the mechanism described in Section 2.1, we specify a supersonic flow at the entrance of the sheath, $x = L_M$. At this position, the electric field is weak, and we can assume that the flow is parallel to the magnetic field lines. Therefore, the boundary condition f_b for the ion distribution function at $x = L_M$ is Maxwellian in $v_\perp$, the velocity perpendicular to the magnetic field, whereas it displays a supersonic mean flow in the parallel velocity ($v_{\parallel 0} = 1.5\ c_s$, for all angles studied below). The initial condition is spatially uniform and equal to f_b. The boundary conditions on the potential are the same as in Section 2.1.

We study the effect of the magnetic field on the sheath for several incidences α (Fig. 1). For $\alpha = 90°$, the magnetic field is perpendicular to the wall and parallel to the electric field. The ion gyro-motion and the parallel flow are decoupled, thus the dynamics is identical to that of an unmagnetized sheath. Fig. 6 shows the ion and electron densities for $\alpha = 40°$ and $10°$. The total sheath thickness increases for small incidences of the magnetic field, while the Debye sheath thickness remains the same, about 5 λ_{De} (Fig. 7). Moreover, the space charge decreases with the incidence of the magnetic field: 30%

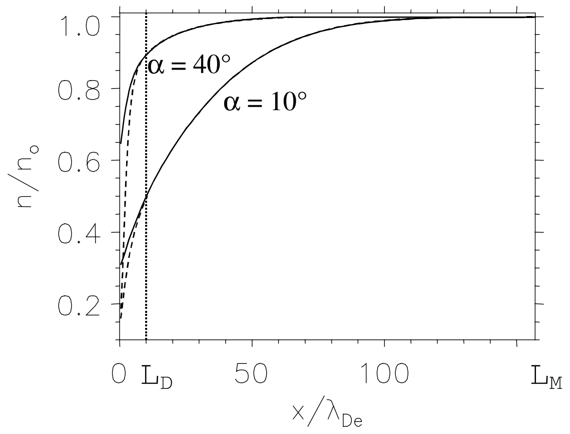

Fig. 6. Ion (solid line) and electron (dashed line) densities (normalized to the bulk plasma density) for different incidences of the magnetic field. L_D: entrance of Debye sheath; L_M: entrance of magnetic presheath.

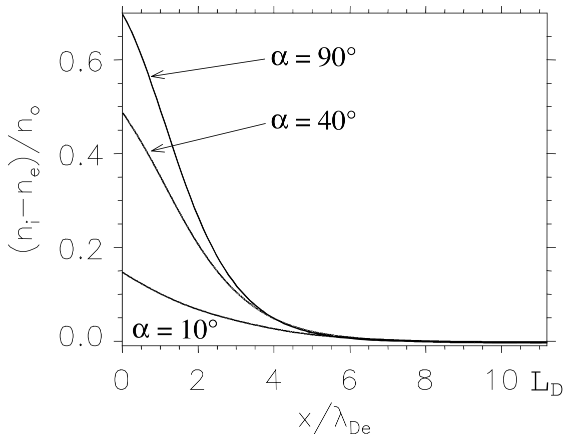

Fig. 7. Space charge in the Debye sheath for different incidences of the magnetic field.

smaller for $\alpha = 40°$ and 70% smaller for $\alpha = 10°$, compared to a case of normal incidence. In the magnetized case, the sheath is, therefore, composed of two parts: the magnetic presheath (between L_D and L_M), which is quasineutral and extends over several ion gyroradii; and the Debye sheath (a few Debye lengths from the wall), where quasineutrality breaks down.

The ion distribution function in the (v_x, v_y) plane perpendicular to the wall and containing the magnetic field is plotted on Fig. 8 for $\alpha = 10°$. We note that the ion distribution function is symmetric with respect to the $v_\parallel$ axis in the bulk plasma, whereas at the wall the axis of approximate symmetry becomes parallel to the v_x axis. The velocity normal to the wall (not shown here), which is set as $v_{x0} = v_{\parallel 0} \sin\alpha = 0.27\ c_s$ at the entrance of the magnetic presheath, becomes equal to the sound speed at $x \approx 5\ \lambda_{De}$ (which roughly corresponds to the Debye sheath entrance, see Figs. 6,7), and is supersonic within the Debye sheath (up to $v_x = 1.4 c_s$ at the wall). This is in agreement with Chodura's result [1], which shows that there are two criteria for the stability of a magnetized sheath. The first criterion is that the flow parallel to **B** must be supersonic at the magnetic presheath entrance: this is automatically satisfied for our boundary condition. The second criterion states that the flow perpendicular to the wall must also be supersonic, but only at the entrance of the electrostatic (Debye) sheath: this has been verified by the results of our simulations. The role of the magnetic field is, therefore, to accelerate the ions in the direction normal to the wall, so that Chodura's second criterion is verified.

3. Conclusion

Several sheath configurations have been studied numerically. First, we have seen that, when the magnetic field is absent, the collisional and ionizing presheath brings the ion flow from subsonic (in the bulk plasma) to supersonic velocity (at the Debye sheath entrance). The

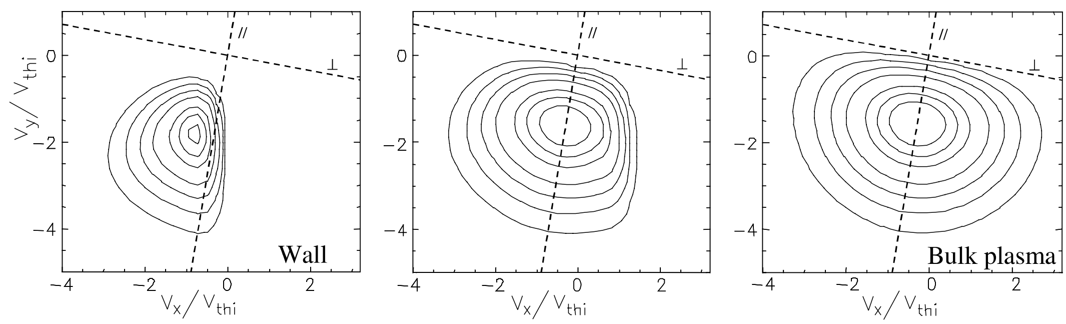

Fig. 8. Contours of constant level of the ion distribution in the (v_x, v_y) plane, at different positions, for $\alpha = 10°$. (Dashed lines correspond to parallel and perpendicular axes to the magnetic field.) Left: $x = 0$, wall; middle: $x = 26.6\ \lambda_{De}$, intermediate position; right: $x = L_M$, bulk plasma.

quasineutrality condition is well-satisfied in the collisional presheath, while a finite space charge builds up in the vicinity of the wall (Debye's sheath). Second, we have investigated the case of a magnetized sheath neglecting collisions and ionization. In this case, the sheath is composed of two parts: the quasineutral magnetic presheath; and the Debye sheath where a space charge can subsist. The use of an Eulerian code has allowed us to obtain accurate results for the ion distribution functions in the phase space. More realistic studies will be needed to analyse the effect of collisions and ionization in the presence of a magnetic field, as well as more complex geometries.

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